HOW TO ....
A PRACTICAL GUIDE TO PSYCHOMETRICS

Calculate the probability that two scores are abnormally different

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1. When you have two scores for one individual on different tests or scales

Use the Z-Score formula for this. To find out whether there is a abnormal difference between two scores (which might of course be subtest scores) calculate the value of:

$$Z_{difference} = \frac{Z_x - Z_y}{\sqrt{2 - 2r_{xy}}}$$

Where Zx = z-score of the score on first test

Zy = z-score of the score on second test

r_{xy} = correlation coefficient between the two tests

The probability of the difference being abnormal can be found by looking up the probability corresponding to the value of $Z_{difference}$ in tables for the normal curve.

Example. Someone obtains a Verbal IQ of 118 and a Performance IQ of 106 on an intelligence test with a mean of 100 and a standard deviation of 15 for all IQ,s. How abnormal is this difference? The correlation between the Verbal Scale and the Performance Scale is .80.

$$Z_{difference} = \frac{(1.20-0.40)}{\sqrt{2-2*0.80}} = \frac{0.80}{\sqrt{2-1.60}} = \frac{0.80}{0.63} \text{ which is } 1.27.$$

A $Z_{difference}$ of 1.27 yields a 2-tail p value of .20. If we adopt the conventional probability levels for deciding that a result is 'significant; then this one is not. We would have to conclude that there is insufficient reason for believing that the difference in score between Verbal IQ is and Performance IQ, because a difference this large or larger occurs in about 20% of examinees.

2. When you have an obtained score and a predicted score for one individual

This is an extension of the situation covered in the last section where you have two scores on two scales for one individual. Except in this case you use one of the scores to predict the score that should be obtained on the other scale. Recall, from the module on correlation that:

$$\hat{Z}y = r_{xy}Zx$$

That is the predicted Z-score on test y is equal to the correlation between test x and test y times the Z-score on test x. Using this information our abnormality formula becomes:

$$Z_{difference} = \frac{Z_y - r_{xy}Z_x}{\sqrt{1-r_{xy}^2}}$$

Where Zx = z-score of the score on first test

Zy = z-score of the score on second test

r_{xy} = correlation coefficient between the two tests

The probability of the difference being abnormal can be found by looking up the probability corresponding to the value of $Z_{difference}$ in tables for the normal curve.

Example. Someone obtains a Full Scale IQ of 108 and a Memory Quotient of 126. Both tests have a mean of 100 and a standard deviation of 15. How abnormal is the difference between the IQ score and the Memory Quotient that would be predicted from that IQ score? The correlation between the two tests is .80.

The Z-score of the IQ test = (108 – 100)/15 = 0.53

The predicted memory quotient Z-score is .80 * .53 = .43

The Z-score of the Memory Quotient = (126 – 100)/15 = 1.73

$$Z_{difference} = \frac{(1.73 - 0.43)}{\sqrt{1-(0.80)^2}} = \frac{1.30}{\sqrt{1-0.64}} = \frac{1.30}{0.60} \text{ which is } 2.17$$

A $Z_{difference}$ of 2.17 yields a 2-tail p value of .03. If we adopt the conventional probability levels for deciding that a result is ‘significant; then this one is. We would have to conclude that there is sufficient reason for believing that the difference in score between the obtained Memory Quotient and the Memory Quotient predicted from the Full Scale IQ is abnormal, because a difference this large or larger occurs in about 3% of examinees.

3. When you have two scores for one individual on the same test or scale
This situation arises when somebody is retested.

The formula is very similar to the two presented above. The difference is that in the above formula we use the correlation between two scales, but in this case we use the reliability co-efficient. In the present case we only have one test, but we have two scores from two different occasions. We use the Z-score on the first occasion to predict a Z-score on the second occasion. Because some tests have a practice effect,

we take this into account, where appropriate, by subtracting the practice effect from the score on the second occasion, before deriving a Z-score. The formula is:

$$Z_{diff} = \frac{Z_{(x_2 - pe)} - r_{x_1 x_2} Z_{x_1}}{\sqrt{1 - r^2_{x_1 x_2}}}$$

Where $Z_{(x_2 - pe)} =$ z-score of (the score on the second occasion – practice effect)

$Z_{x_1} =$ z-score of the score on the first occasion

$r_{x_1 x_2} =$ test-retest reliability coefficient of the test

$pe =$ Practice Effect

$\sigma_x =$ Standard Deviation of the test

Example An 11 year old boy is involved in car accident and receives a head injury. By coincidence a school counsellor had administered an IQ test to the boy 1 month before the accident, as he was being considered for a special program for gifted children. He is re-tested 5 months after the accident using the same test. Before the accident his IQ score was 124. Five months after the accident it was 117. The mean of the test is 100, the standard deviation 15, test-retest reliability is 0.94 and the practice effect over 6 months is 7 points. Is this drop in score abnormally large?

Answer. Applying the formula we get the following values

$$Z_{difference} = \frac{Z_{(117-7)} - 0.94 * Z_{(124)}}{\sqrt{1-(0.94)^2}} = \frac{0.67 - 1.50}{\sqrt{0.116}} = \frac{-0.83}{0.34} = -2.45$$

Consulting tables for the normal distribution we find that about 2 percent of people would show a difference this large or larger than this on a chance basis, and that about 1 percent would show a drop in score as great as this or more. So there is strong evidence to suppose that this child's IQ score has dropped between the two test occasions.