COMBINING SCORES FROM DIFFERENT TESTS

1: COMPOSITES

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Sometimes it is necessary or interesting or useful to combine scores on a number of tests or measures into a meaningful combined score. One way to do this is to form a composite from the tests you are interested in.

But before beginning this topic, it will be worthwhile to refresh our knowledge of covariance.

1.   Covariance

Covariance is defined as:

$$\sigma_{xy} = \frac{\sum xy}{N}$$

where:

σ_{xy} is the symbol for the covariance

$x = X - Mx$

$y = Y - My$

Amongst other things, the covariance is an intrinsic part of the main raw score formula for the correlation coefficient

$$r_{xy} = \frac{\sum xy}{N\sigma_x\sigma_y}$$

It follows from this formula that the covariance is equal to the correlation between *X* and *Y* multiplied by their respective standard deviations. If we multiply both sides of equation (5.2) by $\sigma_x \sigma_y$ we get the following:

$$r_{xy}\sigma_x\sigma_y = \frac{\sum xy}{N}$$

If we are working with z scores (as we will be whenever possible) the standard deviations will both equal 1.0 and thus the covariance of *X* and *Y* will equal the correlation between them.

It also follows that (when both sides of the equation are divided by $\sigma_x\sigma_y$)

$$r_{xy} = \frac{\sigma_{xy}}{\sigma_x\sigma_y}$$

This formula is particularly useful when calculating the correlation between a composite and an outside variable or between two composites.

2.   Why might you want to combine scores

You might have a number of tests measuring different aspects of say, memory, or measuring mental speed, or abstract thinking and would like to combine them into a more global measure of memory or mental speed or abstract thinking

You might want to know whether a subtest score on a test like the WAIS III differed reliably or significantly from scores on a group of other subtests

Or you might have a number of tests purporting to measure the same thing but all of low reliability, and you might want to combine them to obtain a more reliable measurement of the ability or characteristic you are interested in.

3.   How do you combine scores?

You can combine as many scores as you like into a composite score by simply adding the scores on each test together.

But it is preferable and sensible and easier to deal with composites formed by the addition of Z scores on each test

So

Step 1

Convert the score on each test into a Z score

Step 2

Add up the individual Z Score to get the composite score

BUT note that this will **not** give you the Z score on the composite. You will have to do a little more work before converting the scores you have into a Z score for the composite

Step 3

Work out the mean of the composite.

This is mercifully easy.

Simply add together the means of the individual tests making up the composite. This will equal the mean of the composite.

Assuming that you have taken the advice of converting each of the individual component scores into Z scores, you will have, de facto, made the mean score on each of the components equal to zero.

Adding them up will of course give the composite you have formed a mean of zero too.

You can see once again the joy of using Z scores.

Step 4

Work out the standard deviation of the composite

The standard deviation is of course equal to the square root of the variance. So let's work out the variance of the composite. (Remember that we are still using Z-scores)

For this we will need to know the correlations between all of the tests making up the composite.

If our composite consists of only two tests there will be only one correlation between them

If it is made up of three tests then there will be three correlations between them (Test 1 with Test 2, Test 1 with Test 3, and Test 2 with Test 3)

List the correlations and add them up.

The variance of the composite will be equal to :

The sum of the variances of the tests making up the composite plus two times the sum of the correlations between them

Again, rejoice – because we are working in Z scores, the variance of each of the component scores will equal 1.0. So the sum of the variances of the components will equal the number of tests used to make up the composite. And the covariances will all be simply the correlations between the components, So, in Z-score terms, the variance of the composite will be -

The number of components (tests) plus twice the total of the intercorrelations between components.

Example. We have three tests of visual memory A, B and C. The correlations between these tests are as follows:

	A	B	C
A		$r_{AB} = .7$	$r_{AC} = .6$
B	$r_{AB} = .7$		$r_{BC} = .5$
C	$r_{AC} = .6$	$r_{BC} = .5$	

As all of the tests are in Z-scores their variance will be 1.0 in each case

We can now modify the previous table by entering the variance of each test into the diagonal cells – like this

	A	B	C
A	1.0	$r_{AB} = .7$	$r_{AC} = .6$
B	$r_{AB} = .7$	1.0	$r_{BC} = .5$
C	$r_{AC} = .6$	$r_{BC} = .5$	1.0

So the diagonal cells contain the variances of each test, and the off diagonal cells contain the correlations between the tests making up the composite. As we are using Z-scores these correlations are also the covariances

To work out the variance of this visual memory composite all we need to do is add all the values in the table. This gives us a value of 6.6.

Thus we have a new visual memory composite test which has a mean of zero and a variance of 6.6, and hence a standard deviation of the square root of 6.6 which is 2.57

If you are not working with Z-scores the variance of the composite will be:

The sum of the variances of its components plus twice the sum of the covariances between components.

Examples

Two tests of visual memory have been administered. On one the score is a Z of -1.2, on the other the score is a Z of – 1.5. The correlation between the two tests is 0.6. you want to know how poor or good the performance is on the combined tests.

The score on the composite is simply the two Z scores added together, which equals – 2.7

The mean of the composite is of course zero.

What we need to do now is convert the score of – 2.7 into a Z on the distribution of the composite's scores

The variance of the composite will be the sum of the variances of the components, which is 1.0 plus 1.0 which is 2.0 plus two times the correlation between the component tests (2 x 0.6 which is 1.2).

So the variance of the composite is 3.2.

The square root of the variance gives us the standard deviation of the composite, which will be 1.79.

Thus the score converted into a Z score on the composite will equal -2.7/1.79 which equals -1.51

Suppose there had been three tests making up a composite. Z scores on these test were 0.5, 1.2, and 1.0

Correlation between Tests	Correlation coefficient
1 and 2	.5
1 and 3	.6
2 and 3	.7

The variance of the composite will be 3 (the number of tests) plus twice the sum of the correlations between them (2 x 1.8) which gives us a variance of 6.6. Taking the square root of this we get a standard deviation of 2.57.

So the score of 2.7 is 1.05 standard deviations above the mean of scores on the composite.

These Z-scores on composites can be used in formulas for abnormality and reliability of differences in the same way as scores on individual tests.

But of course to see if a difference between a composite score and some other score is reliable we will have to know the reliability of the composite.

4.   The reliability of a composite

You will remember that a reliability coefficient tells us the proportion of variance in a test which is true variance. So it can be found by dividing the true variance by the total variance.

We have just seen how to work out the variance of a composite, but how do we work out the true variance?

Quite easily as it happens!

The true variance of a composite (remember again that we are working in Z-scores) will equal:

The sum of the reliability coefficients of the component tests plus two times the intercorrelations between the component tests.

So, let us suppose that in our last example the reliabilities of the three tests were: 0.7, 0.8, and 0.9.

From what we worked out in the last example we know that the sum of the correlation coefficients was 1.8.

The true variance of the composite will therefore be the sum of the three reliability coefficients (2.4) plus twice the sum of the intercorrelations (2 x 1.8) which equals 6.

To repeat, the reliability of the composite will be its true variance divided by its total variance.

So in this case we have 6 divided by 6.6,

Thus the reliability of our composite test is 0.91, a very respectable figure.

5.   The correlation between a composite and a single outside variable (eg, another test).

 For present purposes an outside variable is a variable that is not one of the components of the composite.

One of the formulas for the correlation coefficient is:

The covariance of two measures divided by the product of their standard deviations

$$r_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

We can use this formula for working out the correlation between a composite and a single outside variable.

Again we will work in Z-scores.

We already know how to work out the variance and standard deviation of a composite, and, of course, as we are working with Z-scores the variance of the single outside variable will equal 1.0

So we simply need to know how to work out the covariance between composites. This will simply be the sum of the correlations of each component with the outside variable.

So let's suppose that we have a composite of three tests of abstract thinking (AT1, AT2 and AT3) and we want to find out the correlation between that composite and IQ.

Variances of, and correlations between tests				
	IQ	AT1	AT2	AT3
IQ	1.0	.6	.8	.5
AT1		1.0	.7	.6
AT2		**.7**	**1.0**	.7
AT3		**.6**	**.7**	1.0

The covariance will be the sum of all the correlations between each component of the composite and IQ, .6 + .8 + .5, which equals 1.9. (The cells used to get the covariance are shaded yellow)

The variance of IQ (shaded blue) will be 1.0 as we are in Z-scores, and the variance of the composite will be found in the usual way, i.e. 3 + .2 x (7 + .6 + .7) which equals 7.0 (The cells used to obtain this variance are shaded grey).

So the correlation between our Abstract Thinking composite and IQ will be:

$$\frac{1.9}{\sqrt{1.0}\sqrt{7.0}} = 0.72$$

6.   The correlation between two composites

Sometimes we might wish to find the correlation between two composites.

We find the covariance between these composites by summing the correlations between each component of the first composite with each component of the second composite. We already know how to work out the variance of each of the composites.

An example will probably help.

Suppose we had a composite of four tests measuring different aspects of visual memory (V1, V2, V3 and V4) and another composite of three tests measuring aspects of auditory memory (A1, A2 and A3).

We want to know the correlation between the two composites.

The table of correlations that we need to arrive at the answer is below. Once again the correlations making up the value of the between covariance are in cells shaded yellow, while the cells needed at calculate the variances of each of the composites are shown in grey and blue.

	V1	V2	V3	V4	A1	A2	A3
V1	1.0	.4	.5	.6	.2	.3	.4
V2	.4	1.0	.7	.5	.5	.4	.3
V3	.5	.7	1.0	.6	.2	.2	.4
V4	.6	.5	.6	1.0	.3	.2	.2
A1					1.0	.6	.7
A2					.6	1.0	.7
A3					.7	.7	1.0

The covariance will equal the sum of all the correlations between each the components of one composite with each of the components of the other (shaded yellow in the diagram). Adding these up gives us a value of 3.6.

The variance of the visual composite will be 4 plus 2 x 3.3, which equals 10.6 (values shown in blue shaded cells). Finally in the grey shaded cells are the values needed to work out the variance of the auditory memory composite 3 plus 2 x 2, which equals 7.

The correlation between the visual memory composite and the auditory memory composite will therefore be:

$$\frac{3.6}{\sqrt{10.6}\sqrt{7.0}} = 0.42$$